

DEPARTMENT OF ECONOMICS

**SINGLE EQUATION MODELS,
CO-INTEGRATION AND THE EXPECTATIONS
HYPOTHESIS OF THE TERM STRUCTURE OF
INTEREST RATES**

Fabrizio Casalin, University of Leicester, UK

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Single Equation Models, Co-Integration and the Expectations Hypothesis of the Term Structure of Interest Rates*

Fabrizio Casalin[†]

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Abstract

The purpose of this paper is twofold. First, by focusing on Single Equation and VECM techniques commonly employed to test for the Expectations Hypothesis of the Term Structure of interest rates (EHTS), it sheds light on the conditions - in terms of the different classes of stochastic processes of the spot and forward rates - that must hold for the EHTS to be valid. In doing so, the existing linkage between the two strands of literature is highlighted. Second, by using kalman filter and maximum likelihood, estimates of a permanent-transitory components model for spot and forward interest rates are carried out. The simple parametric model helps discern the relative contributions of both departures from rational expectation and time varying term premium to the invalidation of the EHTS. Departures from rational expectations turn out to have negligible impact on the rejection of the EHTS. Estimates of the time varying term premia for the short-end of the term structure spectrum are persistent and reasonable in magnitude, and exhibit sign fluctuations.

1 Introduction

The basic idea underlying the Expectations Hypothesis of the Term Structure of interest rates (EHTS) is that, with the exception of a term premium, there should be no expected difference in the returns from holding a long-term bond or rolling over a sequence of short-term bonds. A second version of the EHTS states that, with the exception of a term premium, forward rates are unbiased predictors of future short rates.

The literature has followed different approaches to inspect the EHTS depending on whether the time horizon is focused on either the short or the long run.

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[†]Department of Economics, University of Leicester, University Road, Leicester, LE1 7RH, UK. E-mail: fc49@le.ac.uk.

The studies that focus on the long run exploit the fact that spot and forward interest rates must be driven by a common stochastic trend. Making use of the VECM methodology, they test the necessary conditions imposed by the EHTS; namely that either spot rates at different maturity or forward and spot rates with same maturity are co-integrated with a co-integrating vector such that there exists a one-to-one relationship between them. Examples of this kind of studies are Cuthbertson (1996) and Rossi (1996), who find evidence in favor of the EHTS for the United Kingdom at the short end of the term structure spectrum. Gravelle et al. (1999) reach similar conclusions for the Canadian bond market while other studies such as Hall et al. (1992) and Shea (1992) for the US show little conclusive empirical support (see also Engsted and Tanggaard (1994)). Overall, however, these tests have greater tendency to support the EHTS than those focused on the short run. While the former adopt the VECM methodology, the latter use Single Equation regression methods. In this strand of literature one can distinguish between regressions that employ the *term spread* as a regressor, i.e. the difference between long and short interest rates and regressions which adopt the *forward-spot spread*. The term spread regressions exploit the observation that, with the exception of a term premium, there should be no expected difference in the returns from holding a long-term bond or rolling over a sequence of short term bonds. The forward-spot spread regressions, on the contrary, refer to the idea that with the exception of a term premium, forward rates must be unbiased predictors of future spot rates. Throughout this paper analysis will be carried out that focuses on the forward-spot regressions.

Defining $R_n(t)$ the n -period long rate, $R_m(t)$ the m -period short rates and $F_{n-m}^n(t)$ the m -period forward rate, i.e. the rate at trade date t for a loan between periods $(t + n - m)$ and $(t + n)$, the forward-spot regressions are as follows:

$$F_{n-m}^n(t) - R_m(t + n - m) = \beta_0 + \beta_1(F_{n-m}^n(t) - R_m(t)) + \varepsilon(t + n - m) \quad (1)$$

$$R_m(t + n - m) - R_m(t) = \delta_0 + \delta_1(F_{n-m}^n(t) - R_m(t)) + \varepsilon(t + n - m) \quad (2)$$

where the validity of the EHTS requires that $\beta_1 = 0$ and $\delta_1 = 1$ (see, for instance, Fama (1984), Mishkin (1988) and Fama and Bliss (1987)).

Generally speaking, most of these studies focus on the U.S. term structure. Evidence on tests based on eq.(1) is well summarized by Fama and Bliss (1987) and Mankiw (1986): while short or medium forward-spot yield spreads have strong predictive information for excess yield returns, long term spreads show little or no predictive content. In other words, the EHTS performs poorly at the short end of the maturity spectrum, but improves at longer maturities. Rejections of the EHTS, in turn, reflect systematic forecast errors, time varying term premium, or both (see, for instance, Fama (1984), Mankiw (1986), Tzavalis and Wickens (1997) and Fama and Bliss (1987)). The common shortcoming of both the strands of literature based on VECM and Single Equations is that they

cannot disentangle the relative contribution of departures from rational expectations and time varying term premium to the invalidation of the EHTS. The contribution of this work to the existing literature is, therefore, twofold. First, it aims to bridge the two strands of literature based on co-integration and single-equation models. To the best of my knowledge, this has never been investigated before. So far, in fact, very few works have been written on this topic. Among them Brenner and Kroner (1995) highlighted that one shortcoming of the Single Equation models is that they fail to exploit the existing co-integrating relationships among interest rates. Because of that, empirical tests are impaired by the fact that endogeneity between interest rates is not taken into account. The analysis is conducted working out the moving average representation of the parameters which characterize the regressions based on forward-spot spreads under the hypothesis of co-integration between spot and forward interest rates. Once co-integration is introduced, it becomes possible to separately model departures from rational expectations and time varying term premium. This, in turn, makes it possible to disentangle their contribution to the rejection of the EHTS and to identify their effect on Single Equation and VECM estimates. The estimation of the separate contribution of departures from rational expectations and time varying term premium to the invalidation of the EHTS is the second contribution of this paper. The EHTS is, in fact, a joint hypothesis of rationality and risk neutrality. While a strand of the existing literature has inspected this hypothesis assuming that the rational expectation leg of the theory holds and testing for its second leg - i.e. the presence of time varying term premium - a second strand has inspected the EHTS assuming risk neutrality and testing for its first leg, i.e. rational expectations (see, for instance, Iyer (1997), Lee (1995), Gravelle and Morley (2005), Engle et al. (1987), Hejazi et al. (2000) and Campbell and Shiller (1987)). This work constitutes a first attempt to test the EHTS without taking any a priori stance about the validity of rational expectations and risk neutrality. A quantification of both the rational expectations and term premium effects on EHTS is worked out employing a permanent-transitory components model estimated by mean of kalman filter and maximum likelihood. The same two-component model is then employed to estimate the time varying term premium under the assumption of rational expectations. The resulting estimates are reasonable in magnitude, persistent, and display sign fluctuations. The remainder of the paper is organized as follows. The next section shows that co-integration as well as the effects of departures from rational expectations and time varying term premium on the EHTS can be accounted for by a simple parametric permanent-transitory components model for spot and forward interest rates. In Section 3 this model is employed to work out a moving average formulation for the parameters of the forward-spot regressions. It is shown that such formulation highlights the existing linkage between co-integration and Single Equation models. Moreover, the same formulation helps identify the separate effect of departures from rational expectations and time varying term premium on the parameter itself and, therefore, on the EHTS. A measurement of the two effects is then carried out making use of simulations. Section 4 sheds light on

the existing linkage between co-integration, Single Equation and ECM models. Section 5 discusses the specific version of the permanent-transitory components model employed for estimations. Section 6 presents data and results of the maximum likelihood estimations as well as the tests for the departure from rational expectations, time varying term premium, or both. Section 7 employs simulations to supplement standard diagnostic tests in evaluating the model. Section 8 presents estimates of time varying term premia. Finally, conclusions are presented in Section 9.

2 Co-Integration and EHTS

One problem that has hampered empirical works is that there are many different versions of the EHTS, as emphasized by Campbell and Shiller (1991) and others. Throughout this paper the version that posits that the m -period forward rate $F_{n-m}^n(t)$ should equal the expected return at trade date t for an m -period asset between periods $(t+n-m)$ and $(t+n)$ is considered. Although the occurrence of co-integration in the term structure of interest rates is, by now, a consolidated fact, related literature has always overlooked the linkage between co-integration and Single Equation models. To shed light on such linkage, co-integration is introduced in the analysis exploiting Stock and Watson's (1993) observation that co-integrated variables can be expressed as a linear combination of I(1) common stochastic trends and I(0) components.

Applying this result to the term structure, one would expect the presence of a single nonstationary common factor in yields of different maturity. Denoting the I(1) common factor by $W(t)$, a simple representation of how it links the yields curve is given by

$$R_1(t) = A(1, t) + b_1 W(t)$$

$$R_2(t) = A(2, t) + b_2 W(t)$$

.....

$$R_n(t) = A(n, t) + b_n W(t)$$

in which the $A(i, t)$ are the I(0) components. Being $W(t)$ I(1) and $A(i, t)$ I(0), the long-run movements in each yields are mainly driven by movements in the common factor.

The assertion that a common driving force underlies the time series behavior of each yield to maturity is not new in the literature on the term structure. Cox et al. (1985), for instance, build a continuous time general equilibrium model of real yield to maturity in which the instantaneous interest rate is common to all yields. In the discrete time model developed by Campbell and Shiller (1987) it is emphasized how there is only one nonstationary I(1) common driving force which can be interpreted as something exogenous to the system of the term structure such as inflation or measures of monetary growth (see also Campbell and Shiller (1991), Cuthbertson (1996), Engle (2002), Engsted and Tanggaard

(1994), Hall et al. (1992) and Gravelle et al. (1999)).

Although the above representation regards spot interest rates, a similar framework can be adopted also for forward and spot interest rates, and implies that forward and expected spot rates evolve according to a common stochastic trend. Let us define the following stochastic processes:

$$F_{n-m}^n(t) = \mu_{F_{n-m}^n}(t) + x_{F_{n-m}^n}(t), \quad \mu_{F_{n-m}^n}(t) = \mu_{F_{n-m}^n}(t-1) + \epsilon_{F_{n-m}^n}(t) \quad (3)$$

$$R_m(t) = \mu_{R_m}(t) + x_{R_m}(t), \quad \mu_{R_m}(t) = \mu_{R_m}(t-1) + \epsilon_{R_m}(t) \quad (4)$$

where $\mu_{F_{n-m}^n}(t)$ and $\mu_{R_m}(t)$ are random walk processes while $\epsilon_{F_{n-m}^n}(t)$ and $\epsilon_{R_m}(t)$ are independently distributed white noise disturbances. In line with the Stock and Watson's (1993) representation, no restrictions are imposed on the stochastic properties of the processes $x_{F_{n-m}^n}(t)$, $x_{R_m}(t)$ beyond being ARMA stationary. To strike a balance between flexibility and model parsimony these transitory deviations from the stochastic trends are represented by the following vector ARMA process:

$$\begin{bmatrix} \phi(L)_{FF} & \phi(L)_{FR} \\ \phi(L)_{RF} & \phi(L)_{RR} \end{bmatrix} \begin{bmatrix} x_{F_{n-m}^n}(t) \\ x_{R_m}(t) \end{bmatrix} = \begin{bmatrix} \theta(L)_{FF} & \theta(L)_{FR} \\ \theta(L)_{RF} & \theta(L)_{RR} \end{bmatrix} \begin{bmatrix} \epsilon_{F_{n-m}^n}(t) \\ \epsilon_{R_m}(t) \end{bmatrix} \quad (5)$$

with

$$\begin{bmatrix} \epsilon_{F_{n-m}^n}(t) \\ \epsilon_{R_m}(t) \end{bmatrix} \sim iid N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_F^2 & \sigma_{F,R} \\ \sigma_{R,F} & \sigma_R^2 \end{bmatrix} \right) \quad (6)$$

where the $\phi(L)$'s and $\theta(L)$'s are polynomials in the lag operator L .

The condition for co-integration to occur between forward and future spot interest rates can be worked out imposing stationarity in the following linear combination:

$$\begin{aligned} & \lambda_1 \cdot F_{n-m}^n(t) + \lambda_2 \cdot R_m(t+n-m) = \\ & = (\lambda_1 \cdot \mu_{F_{n-m}^n}(t) + \lambda_2 \cdot \mu_{R_m}(t+n-m)) + \\ & + (\lambda_1 \cdot x_{F_{n-m}^n}(t) + \lambda_2 \cdot x_{R_m}(t+n-m)). \end{aligned} \quad (7)$$

From eq.(7), the necessary and sufficient condition for $R_m(t+n-m)$ and $F_{n-m}^n(t)$ to be co-integrated is:

$$\mu_{R_m}(t+n-m) = -\frac{\lambda_1}{\lambda_2} \cdot \mu_{F_{n-m}^n}(t) = k_{1,2} \cdot \mu^*(t) \quad (8)$$

where $k_{1,2} = -(\lambda_1/\lambda_2)$ is a constant value. Thus, when co-integration occurs, the forward and the future spot rate must evolve according to the following stochastic processes:

$$F_{n-m}^n(t) = \mu^*(t) + x_{F_{n-m}^n}(t) \quad (9)$$

$$R_m(t+n-m) = k_{1,2} \cdot \mu^*(t) + x_{R_m}(t+n-m). \quad (10)$$

$$\mu^*(t) = \mu^*(t-1) + \epsilon(t). \quad (11)$$

In other words, both the forward and the future spot rate must be driven by the same I(1) stochastic trend $\mu^*(t)$. In turn, eq.(8) shows that this stochastic trend at time t , captures the driving forces underlying future spot rate $R_m(t+n-m)$. Such trend can be thought of as the "fundamental" or "long run equilibrium" which drives the term structure. Taking expectations at time t , eqs.(9)-(10) can be rewritten as:

$$F_{n-m}^n(t) = \mu^*(t) + x_{F_{n-m}^n}(t) \quad (12)$$

$$E_t[R_m(t+n-m)] = k_{1,2} \cdot \mu^*(t) + E_t[x_{R_m}(t+n-m)]. \quad (13)$$

From eqs.(12)-(13) it can be clearly seen that both the forward and the future spot rate are determined by two different components; the former is the common trend which evolves according to an I(1) process, while the latter can be interpreted as a (stationary) "omnibus" terms which encapsulates all the residual forces which affect the two rates. Both $x_{F_{n-m}^n}(t)$ and $x_{R_m}(t)$, being stationary processes, are of second order in importance with respect to the stochastic trend. The next section will show that the term premium formulation depends on both these stationary processes. For instance, setting $n=6$ and $m=3$ (i.e. taking into consideration six and three month spot interest rates) the stochastic processes (12)-(13) become:¹

$$F_3^6(t) = \mu^*(t) + x_{F_3^6}(t) \quad (14)$$

$$E_t[R_3(t+3)] = k_{1,2} \cdot \mu^*(t) + E_t[x_{R_3}(t+3)] \quad (15)$$

$$\mu^*(t) = \mu^*(t-1) + \epsilon(t). \quad (16)$$

Thus, the introduction of co-integration in the analysis helps discern the driving forces of the forward and spot rates into two different components. The former is related to expectations of nonstationary I(1) fundamentals which drive the two rates, while the latter consists of stationary terms which will be shown to be related to the term premium. As it will be shown later, this will help disentangle the relative contribution of agents' expectations and time varying term premium to the departure from the EHTS. More specifically, the rational expectations leg of the EHTS in eqs.(12)-(11) is modeled through the ratio $k_{1,2}$. When $k_{1,2}$ equals 1, then expectations about future fundamentals are formed correctly, i.e. the forward rate at time t will match, on average, the future spot rate. When, however, it differs from 1 then expectations of future values of the spot rate turn out to be systematically wrong. The more $k_{1,2}$ departs from 1, the stronger the departure from rational expectations. The time varying term premium leg of the EHTS will be modeled in the next section.

¹In the stochastic processes (12)-(11) the components $W(t)$ and $A(\cdot, t)$ are defined, respectively, by $\mu^*(t)$ and the stationary processes $x_{F_{n-m}^n}(t)$ and $x_{R_m}(t+n-m)$.

3 The Relationship between Co-Integration and Single Equation Models

3.1 Single Equation Models

Tests for the EHTS can be carried out making use of both eqs.(1) and (2). Since the regressor in both regressions is the same variable, and the sum of the regressands equals the regressor, the two equations are entirely complementary (i.e. $\beta_1 = 1 - \delta_1$). As such, analysis carried out employing either eq.(1) or (2) are totally equivalent.

Exploiting the notation previously set out, let Δ^{n-m} be the $(n-m)$ -period difference operator ($\Delta^{n-m}R_m(t+n-m) \equiv R_m(t+n-m) - R_m(t)$ with $\Delta^1 \equiv \Delta$), $TP_{n-m}^n(t) = F_{n-m}^n(t) - E_t[R_m(t+n-m)]$ be the term premium, and $fp_{n-m}^n(t) = F_{n-m}^n(t) - R_m(t)$ be the $(n-m)$ -period forward premium. Given the above notation, it is possible to write the formulation of β_1 as follows:

$$\beta_1 = \frac{Cov[TP_{n-m}^n(t); fp_{n-m}^n(t)]}{Var[fp_{n-m}^n(t)]}. \quad (17)$$

The dataset employed begins in January 1964 and extends to May 2000 for the spot interest rates $R_3(t)$, $R_6(t)$ and $R_{12}(t)$, i.e. respectively the three, six and twelve months Treasury Bill rates (average auctions). These data are taken from the FRED database at the Federal Reserve Bank of St.Louis. The forward interest rates $F_3^6(t)$ and $F_6^{12}(t)$ are the rates implicit in the yield curve extracted using the three, six and twelve month spot rates.²

Table 1 reports the OLS estimates of eq.(1) where the spreads employed are the three and six-month spot and forward interest rates.³ The two slopes are 0.671 and 0.889. Both of them are more than 2.0 standard errors from 0.0 while they are within one standard error of 1.0. These results, in turn, suggest that the EHTS is rejected. Rejections of the EHTS, in turn, can be caused by time varying term premium, departures from rational expectations, or both.

3.2 Co-integration and Single Equation Models

Exploiting the parametric representation of $F_{n-m}^n(t)$ and $R_m(t)$, one can express these in terms of their specific (idiosyncratic) disturbance terms and the disturbance terms of the common stochastic trend. In doing so, it becomes possible to formulate the parameter β_1 in such a way that the existing linkage between co-integration and Single Equation models is highlighted.

In order to strike a balance between flexibility and model parsimony, the analysis

²The length of the period under analysis is limited by the fact that the FRED database makes available these data only up to May 2000.

³The standard errors of the regression coefficients are adjusted for possible heteroscedasticity and for serial correlation induced by the overlap of monthly observations on annual return making use of the Newey and West's (1987) technique.

Table 1: Forward premium regressions (OLS estimates of eq.(1)).

	t-ratio			t-ratio		
	$\hat{\alpha}_1$	s.e.	$H_0 : \alpha_1 = 0$	$\hat{\beta}_1$	s.e.	$H_0 : \beta_1 = 0$
n=6 m=3	0.030	0.032	0.936	0.671	0.239	2.805
n=12 m=6	-0.002	0.042	-0.049	0.899	0.240	3.733

Notes: Corrections for serial correlation in the regression errors employ Newey and West (1987) with truncation lag of the Bartlett window equal to 3.

is carried out by examining the special case where the transient components follow univariate AR(1) processes with contemporaneously correlated innovations - a simplification which helps the interpretation of analytic formulae.⁴ Derivation of the parameter β_1 is therefore worked out setting $\phi_{RR}(L) = 1 - \phi_R(L)$, $\phi_{FF}(L) = 1 - \phi_F(L)$, $\theta_{RR}(L) = \phi_{FF}(L) = 1$, and $\phi_{FR}(L) = \phi_{RF}(L) = \theta_{FR}(L) = \theta_{RF}(L) = 0$ in eq.(5).

Setting m=3 and n=6 and assuming co-integration as defined by eqs.(9)-(10), the random walk-AR(1) model implies the following population value of β_1 (note that the same intuition carries over to m=6 and n=12 as well):

$$\beta_1 = \frac{(1 - k_{1,2})\sigma_\mu^2[(1 - k_{1,2})(t - 3) + 3] + \frac{\sigma_F^2}{1 - \phi_F^2} + \frac{\phi_R^3 \sigma_R^2}{1 - \phi_R^2} - \frac{(1 + \phi_R^3)\sigma_{R,F}}{1 - \phi_F \phi_R}}{\sigma_\mu^2[(1 - k_{1,2})^2(t - 3) + 3] + \frac{\sigma_F^2}{1 - \phi_F^2} + \frac{\sigma_R^2}{1 - \phi_R^2} - \frac{2\sigma_{R,F}}{1 - \phi_F \phi_R}} \quad (18)$$

where σ_μ^2 is the variances of the common stochastic trend, ϕ_F and ϕ_R are, respectively, the autoregressive coefficients for the stationary stochastic processes $x_{F_3^6}(t)$ and $x_{R_3}(t)$, and t is the number of observations employed to estimate eq.(1).

Eq.(18) sheds light on the linkage between the co-integrating relationship occurring between forward and future spot interest rate and the outcomes of tests for the EHTS when Single Equation models are employed. More specifically, the formula shows that $\beta_1 = 0$ is a very special case. A particular set of restrictions that produces this result is for both forward and future spot interest rates to be generated by a common random walk plus noise where the noise term has contemporaneous correlation equal to the ratio of their standard deviations (i.e. $k_{1,2} = 1$, $\phi_R = \phi_F = 0$ and $\sigma_{RF} = \sigma_F^2$). This implies that the term premium will evolve as an i.i.d. process with variance σ_F^2 and its covariance with the

⁴Later in Section 5 it will be shown that the absence of contemporaneously correlated innovations is a necessary restriction that must be imposed to insure that the estimated econometric model is identified.

expected change in spot rates will be $-\sigma_F^2$. These results are fully consistent with the findings of Hai et al. (1997).⁵ Along with this set of conditions, a more general set of conditions consistent with the EHTS requires both forward and future spot rates to be driven by the same stochastic trend with the absence of transitory processes (i.e. $k_{1,2} = 1$ and $\sigma_R^2 = \sigma_F^2 = 0$). Focussing on the framework defined by eqs.(9)-(10), it becomes possible to measure the separate effects of departures from rational expectations and time varying term premium on β_1 . While the rational expectations leg of the EHTS is modeled by means of the coefficient $k_{1,2}$, to understand the role played by the term premium one needs first to work out its expression.

3.3 The Time Varying Term Premium

The time varying term premium leg of the EHTS can be modeled recalling that the term premium is given by the difference between forward and expectations (formed at time t) of future spot interest rate. Setting $n=6$ and $m=3$, one can work out the general formulation of the first and second moment of the term premium as follows (note that the same intuition carries over to $m=6$ and $n=12$ as well):

$$\begin{aligned} TP_3^6(t) &= F_3^6(t) - E_t[R_3(t+3)] = \\ &= (1 - k_{1,2})\mu^*(t) + x_{F_3^6}(t) - \phi_R^3 x_{R_3}(t) \end{aligned} \quad (19)$$

$$Var_t[TP_3^6(t)] = (1 - k_{1,2})^2 t \sigma_\mu^2 + \frac{\sigma_f^2}{1 - \phi_f^2} + \phi_R^3 [\phi_R^3 \frac{\sigma_R^2}{1 - \phi_R^2} - \frac{2\sigma_{R,F}}{1 - \phi_R \phi_F}]. \quad (20)$$

In line with the existing literature, the canonic definition of term premium can be worked out assuming rational expectations. According to eqs.(19)-(20), when $k_{1,2} = 1$ is imposed the term premium evolves as a stationary process with volatility and persistency characterized, respectively, by the parameters σ_F^2 , σ_R^2 and $\sigma_{F,R}$ and ϕ_F and ϕ_R . These parameters, in turn, are among those which characterize the moving average representation of β_1 in eq.(18).

The EHTS, however, can be rejected because of both time varying term premium and departures from rational expectations. When the assumption of rational expectations is relaxed (i.e. when $k_{1,2}$ departs from 1), this introduces a unit root in stochastic process of eqs.(19)-(20) which blurs the stochastic properties of the term premium.⁶ In Section 6, however, it will be shown that it is still possible to detect the stochastic properties of the term premium also in case of departures from rational expectations by making use of eq.(19) and kalman filtering estimation.

⁵Hai et al. (1997) conduct a similar analysis for forward and spot exchange rates.

⁶From eqs.(19)-(20) it can be seen that the more pronounced is the departure from rational expectations the stronger is the presence of the unit root in the stochastic process.

Moreover, the above formulation under the general case of non rational expectations and time varying term premium, highlights an important shortcoming of the literature on term premium. A large body of this literature, in fact, has inspected the properties of the term premium assuming rational expectation (see, for instance, Lee (1995), Iyer (1997), Engle et al. (1987), Hejazi et al. (2000) and Gravelle and Morley (2005)). In doing so, the approach followed was to work out the term premium as the difference between future spot and forward rate. To the extent that departures from rational expectations occur, such difference will contain a unit root which blurs the real stochastic properties of the term premium. For instance, the findings of Iyer (1997), and Gravelle and Morley (2005), who show that the time varying term premium is actually an I(1) process, would suggest departures from the rational expectations hypothesis.⁷

3.4 Parameters and the EHTS

While departures from rational expectations and time varying term premium have been modeled in the previous paragraphs, this paragraph inspects their relative contribution to the invalidation of the EHTS. Throughout the analysis, the baseline condition consistent with the EHTS is assumed to be $k_{1,2} = 1$, $\sigma_R^2 = 0$ and $\sigma_F^2 = 0$. Such a condition implies that the parameter β_1 is equal to 0.

It can be shown that σ_μ^2 , the variability of the common stochastic trend, plays a crucial role in the determination of the values of β_1 . In fact, when both the forward and future spot interest rates are driven by the common stochastic trend (i.e. when both the ratios σ_F^2/σ_μ^2 and σ_R^2/σ_μ^2 tend to zero), the parameter β_1 converges to

$$\lim_{\frac{\sigma_F^2}{\sigma_\mu^2}, \frac{\sigma_R^2}{\sigma_\mu^2} \rightarrow 0} \beta_1 = \frac{(1 - k_{1,2})[(1 - k_{1,2})(t - 3) + 3]}{[(1 - k_{1,2})^2(t - 3) + 3]}. \quad (21)$$

From eq.(21) it can be seen that, when the negligibility of the time varying term premium is guaranteed by the fact that $\sigma_F^2/\sigma_\mu^2 \rightarrow 0$ and $\sigma_R^2/\sigma_\mu^2 \rightarrow 0$, then the EHTS holds as long as $k_{1,2} = 1$. More generally, the stronger the co-integrating relationship, the more β_1 will be dependent on the parameter $k_{1,2}$ and the less it will be on σ_F^2 and σ_R^2 , which are the two parameters which govern the time varying term premium. When, on the other hand, co-integration is weak, the effect of σ_F^2 and σ_R^2 becomes relevant along with the effect of $k_{1,2}$. It follows that whenever forward and future spot rates are co-integrated processes, a necessary condition for the EHTS to hold is that a one-to-one relationship between the two processes occurs. When the co-integrating relationship is strong enough to absorb the stochastic properties of the term premium, then the above necessary condition turns into a sufficient condition.

⁷For moderate departures from the rational expectations hypothesis (i.e. for values of $k_{1,2} \simeq 1$), however, eqs.(19)-(20) remain consistent with the common finding of stationary but highly persistent term premium (see, for example, Engle et al. (1987), Gravelle et al. (1999), Engsted and Tanggaard (1994)).

The EHTS, on the other hand, is rejected whenever the two processes are co-integrated but co-move not into a one-to-one relationship or the two processes are not co-integrated. In fact, according to eq.(21), whenever $k_{1,2}$ departs from 1 and the common stochastic trend is responsible for most of the variability in both the forward and spot rates, then β_1 will tend to 1^- . This can be clearly seen from Figure 1 which depicts the values of β_1 depending on $k_{1,2}$ and σ_μ^2 .⁸ The figure shows that the closer $k_{1,2}$ to 1 and the higher σ_μ^2 (the lower the ratios σ_R^2/σ_μ^2 and σ_F^2/σ_μ^2) the more β_1 approaches zero, while the more $k_{1,2}$ departs from 1 the more β_1 approaches 1^- . From Figure 1 it can also be seen that, when co-integration occurs, the parameter β_1 will span the interval $(0;1)$.⁹ On the other hand, when spot and forward rates are driven by I(1) processes not co-integrated, the moving average formulation of the parameter β_1 is as follows:

$$\beta_1 = \frac{t[\sigma_F^2 + \sigma_R^2 - 2\sigma_{R,F}]}{t[\sigma_F^2 + \sigma_R^2 - 2\sigma_{R,F}]} = 1. \quad (22)$$

The above formulation shows that the population value of β_1 assumes values equal to 1 irrespective of the stochastic properties of both forward and spot rates. This result, in turn, leads to strong rejection of the EHTS. From this and the above results it follows that a necessary condition for the EHTS to hold, when forward and spot rate are I(1) processes, is that they must be co-integrated with $k_{1,2} = 1$.

In case of absence of the common stochastic trend (i.e. when $\sigma_\mu^2 = 0$) the parameter β_1 assumes the following formulation:

$$\beta_1 = \frac{\frac{\sigma_F^2}{1-\phi_F^2} + \phi_R^3 \frac{\sigma_R^2}{1-\phi_R^2} - (1 + \phi_R^3) \frac{\sigma_{R,F}}{1-\phi_F\phi_R}}{\frac{\sigma_F^2}{1-\phi_F^2} + \frac{\sigma_R^2}{1-\phi_R^2} - 2 \frac{\sigma_{R,F}}{1-\phi_F\phi_R}}. \quad (23)$$

The case of absence of the common stochastic trend depicts a case in which spot and forward interest rates are stationary processes which evolve without any common factor. In this case it can be shown that the parameter β_1 takes values close to 1. It follows that the absence of a common stochastic trend in forward and future spot interest rates is a condition not consistent with the EHTS.¹⁰

⁸Since Section 6 conducts an empirical analysis employing monthly observations for a period of 36 years, the values of β_1 in Figure 1 are worked out employing eq.(21) and setting $t = 432$.

⁹From the above analysis it can also be shown that the stronger the stochastic trend, the higher the degree of convergence of β_1 toward 0 and 1^- . Moreover, from Figure 1 appears that β_1 is slightly asymmetrical with respect to $k_{1,2} = 1$.

¹⁰For instance, when $\sigma_\mu^2 = 0$ departures from the EHTS exacerbate as $\phi_R \rightarrow 1^-$, in fact:

$$\lim_{\phi_R \rightarrow 1^-} \beta_1 = 1^-.$$

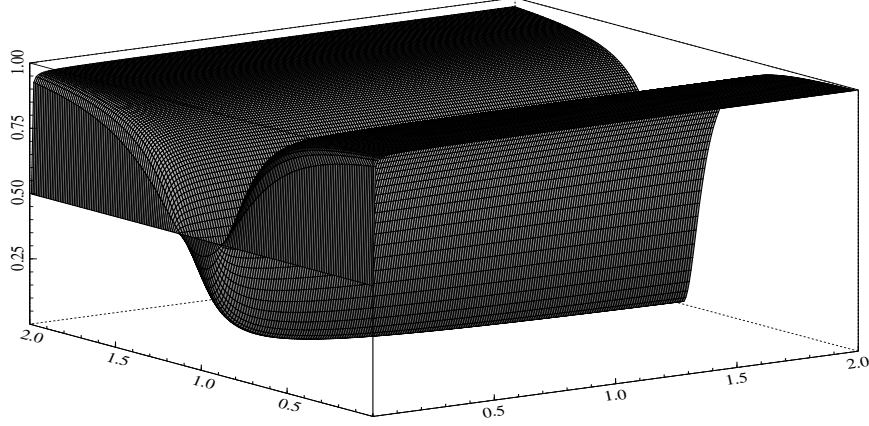


Figure 1: Values of β_1 (vertical axis) dependent on $k_{1,2}$ (longitudinal axis) and σ_μ^2 (latitudinal axis) when $\sigma_R^2 = 0.2$, $\sigma_F^2 = 0.2$ and $t=432$. Both $k_{1,2}$ and σ_μ^2 span the interval $[0;2]$.

3.5 Parameters under the hypothesis of rational expectations

The EHTS is a joint hypothesis of rationality and risk neutrality. A large part of the literature has inspected such hypothesis assuming that the rational expectation leg of the theory holds, and testing for its second leg - i.e. the presence of time varying term premium. In this section, in line with this strand of literature, the assumption of rational expectations is imposed setting $k_{1,2} = 1$ in the stochastic processes defined by eqs.(9)-(10). Thus, except for transitory departures from the long run equilibrium, forward and future spot interest rates co-move into a one-to-one relationship. In other words, agents form expectations rationally and any (temporary) departures from the co-integrating equilibrium is interpreted as time varying term premium. As a result, the parameter β_1 boils down to:

$$\beta_1 = \frac{\frac{\sigma_F^2}{1-\phi_F^2} + \phi_R^3 \frac{\sigma_R^2}{1-\phi_R^2} - (1 + \phi_R^3) \frac{\sigma_{R,F}}{1-\phi_F\phi_R}}{3\sigma_\mu^2 + \frac{\sigma_F^2}{1-\phi_F^2} + \frac{\sigma_R^2}{1-\phi_R^2} - 2 \frac{\sigma_{R,F}}{1-\phi_F\phi_R}}. \quad (24)$$

Eq.(24) shows that, under rational expectations, β_1 can take values within the interval $[0,1]$.¹¹ Its values are shown to depend on the parameters which characterize the volatility (i.e. σ_F^2 , σ_R^2 and $\sigma_{F,R}$) and persistency (i.e. ϕ_F and ϕ_R)

¹¹Section 3.2 has shown that under fairly general conditions (which encompass rational expectations) the parameter β_1 must take values within the interval $[0;1]$.

of the time varying term premium. As expected, the value of β_1 consistent with the EHTS requires absence of time varying term premium. On the contrary, the more volatile and the more persistent is the term premium, the more β_1 will depart from 0 and the stronger will be the evidence against the EHTS. Moreover, eq.(24) shows that also in presence of rational expectations the common stochastic trend plays an important role in favor of the validation of the EHTS. More specifically, the stronger the common stochastic trend (i.e. the lower the ratios σ_R^2/σ_μ^2 and σ_F^2/σ_μ^2), the more the effects of the time varying term premium are attenuated and the more β_1 will approach values close to zero.

A second interesting result can be drawn from the findings of Evans and Lewis (1994) who show that, in presence of rational expectations and time varying term premium correlated with the forward-spot yield spread, the estimates of the slope coefficient of eq.(1) departs from zero and takes values proportional to the covariance between term premium and forward-spot yield.¹² Recalling, from the previous paragraph, that when co-integration occurs, the parameter β_1 must span the interval $[0;1]$, it can be shown that

$$0 < Cov[TP_{n-m}^n(t); F_{n-m}^n(t) - R_m(t)] < Var[F_{n-m}^n(t) - R_m(t)]. \quad (25)$$

As a result, when rational expectations and co-integration occur and the time varying term premium is to blame for rejecting the null of EHTS, the term premium itself must be positively correlated with the forward-spot yield with upper bound given by the variance of the forward-spot yield.

3.6 Simulations

To further inspect the evidence reported in paragraphs 3.2 and 3.5 one can conduct Monte Carlo experiments to generate the empirical distribution of the estimates of β_1 under different assumptions. Specifically, for the stochastic processes defined by eqs.(9)-(10), a sequence of normal shocks $(\epsilon_{F_{n-m}^n}(t), \epsilon_{R_m}(t), \epsilon(t))$ equal in length to 1,000 observations is generated. With these shocks, series on $F_{n-m}^n(t)$ and $R_m(t)$ are constructed and employed to carry out estimations of eq.(1). This entire procedure is repeated 10,000 times to form the empirical distributions of the estimator $\hat{\beta}_1$. Since simulations for $n=6$ and $m=3$ as well as for $n=12$ and $m=6$ provide similar evidence, to save space only the former are set out.

The first Monte Carlo experiment is conducted under the assumption of EHTS. For the EHTS to be true the following two combinations of parameters must be imposed into the data generating process: either $\phi_F = \phi_R = 0$, $\sigma_{R,F} = \sigma_F^2$ and $k_{1,2} = 1$ or $\sigma_F^2 = \sigma_R^2 = 0$ and $k_{1,2} = 1$.¹³ Table 2 displays the lower 2.5, 50 and 97.5 percentiles of the empirical distributions for $\hat{\beta}_1$, its asymptotic t -ratios as well as Jarque-Bera tests for normality. As in Table 1, the asymptotic

¹²This result was worked out under fairly general assumptions which do not require co-integration.

¹³See paragraph 3.2.

t -ratios are constructed under the hypothesis that the slope coefficient is 0. As expected, under EHTS the estimates $\hat{\beta}_1$ are shown to be normally distributed with mean zero and never statistically different from zero.

The second experiment assumes that the EHTS is rejected because of the presence of time varying term premium while the validity of the rational expectations hypothesis is not violated. Recalling eq.(19), the stochastic properties of the time varying term premium are modeled assigning different values to σ_F^2 , σ_R^2 , whereas higher values of both the parameters implies more volatile term premium.¹⁴ Thus, simulations are generated feeding the data generating process with different combinations of σ_F^2 , σ_R^2 and imposing $k_{1,2} = 1$. Table 3 displays the lower 2.5, 50 and 97.5 percentiles of the empirical distribution for $\hat{\beta}_1$ and its asymptotic t -ratio. The table shows that the stochastic properties of $\hat{\beta}_1$ depends on the importance of the term premium relative to the strength of the common stochastic trend (i.e. on the relative importance of the ratios σ_F^2/σ_μ^2 and σ_R^2/σ_μ^2). More specifically, the larger σ_F^2 and σ_R^2 relative to σ_μ^2 , the more the empirical distribution departs from zero shifting rightward. This, in turn, implies rejections of the null of EHTS as the median of the asymptotic t -ratios show. On the contrary, the stronger the stochastic trend relative to the term premium, the more the distribution shifts toward zero. Moreover, the Jarque-Bera test computed for all the empirical distributions fails to reject the null of normality. Thus, when the presence of time varying term premium is to blame for the rejection of the EHTS, the estimator $\hat{\beta}_1$ remains normally distributed.

The third Monte Carlo experiment is conducted under the assumption that the only cause of rejection of the EHTS is departures from the rational expectations hypothesis. As it was shown in Section 2, departures from rational expectations can be modeled assigning to $k_{1,2}$ values different from 1. The absence of time varying term premium, instead, is modeled setting both σ_F^2 and σ_R^2 equal to zero. Table 4 shows that the stochastic properties of $\hat{\beta}_1$ are strongly depend on the values of $k_{1,2}$. In fact, the more $k_{1,2}$ departs from 1 (i.e. the stronger the departure from the rational expectations hypothesis) the more the probability distribution of $\hat{\beta}_1$ departs from zero and shifts rightward.¹⁵ Moreover, as the data generating process departs from rational expectations, the empirical distributions of $\hat{\beta}_1$ degenerate to non normal distributions. This last result can be explained noting that under the hypothesis of co-integration between forward and future spot interest rate, the term $F_{n-m}^n(t) - R_m(t + n - m)$ is stationary as long as the co-integrating vector is $[1 \quad -1]$. Thus, to the extent that $k_{1,2}$ departs from 1, the L.H.S. of eq.(1) contains a unit root and the equation itself becomes a spurious regression. This last point will be further clarified in the next section.

¹⁴To simplify the analysis the remaining parameters which govern the term premium are set equal to $\rho_{R,F} = 0.5$, $\phi_F = 0.85$ and $\phi_R = 0.85$ while the variance of the common stochastic trend is set equal to 0.02.

¹⁵As it was shown in par. 3.2, the upper bound for the population value of β_1 when rational expectations do not hold is 1^- .

Table 2: Stochastic properties of the empirical distribution of $\hat{\beta}_1$ under EHTS.

	Slope coefficient			Asymptotic t-ratio		
	2.5%	median	97.5%	2.5%	median	97.5%
$\phi_F = \phi_R = 0, \sigma_{R,F} = \sigma_F^2$						
$\hat{\beta}_1$	-0.0005	0.0000	0.0005	-0.052	0.004	0.061
JB	0.844 (0.655)			0.817 (0.664)		
$\sigma_F^2 = \sigma_R^2 = 0$						
$\hat{\beta}_1$	-0.0004	0.000	0.0005	-0.052	0.004	0.061
JB	0.058 (0.971)			0.101 (0.950)		

Notes: The data-generating mechanism is the random walk vector ARMA components model fitted to spot and 3-month forward interest rate with $\sigma_\mu^2 = 0.02$, $k_{1,2} = 1$ and $\rho_{R,F} = 0.5$.

Table 3: Stochastic properties of the empirical distribution of $\hat{\beta}_1$ under rational expectations and time varying term premium.

	Slope coefficient			Asymptotic t-ratio		
	2.5%	median	97.5%	2.5%	median	97.5%
$\sigma_F^2 = 0, \sigma_R^2 = 0$ and $\sigma_\mu^2 = 0.02$						
$\hat{\beta}_1$	-0.0004	0.000	0.0005	-0.052	0.004	0.061
JB	0.058 (0.971)			0.101 (0.950)		
$\sigma_F^2 = 0.01, \sigma_R^2 = 0.01$ and $\sigma_\mu^2 = 0.02$						
$\hat{\beta}_1$	0.649	0.711	0.770	1.332	1.517	1.714
JB	2.242 (0.326)			4.547 (0.103)		
$\sigma_F^2 = 0.02, \sigma_R^2 = 0.02$ and $\sigma_\mu^2 = 0.02$						
$\hat{\beta}_1$	0.740	0.792	0.839	1.640	1.858	2.094
JB	4.600 (0.100)			5.230 (0.073)		
$\sigma_F^2 = 0.03, \sigma_R^2 = 0.03$ and $\sigma_\mu^2 = 0.02$						
$\hat{\beta}_1$	0.776	0.823	0.864	1.802	2.035	2.290
JB	6.005 (0.049)			5.563 (0.061)		

Notes: The data-generating mechanism is the random walk vector ARMA components model fitted to spot and 3-month forward interest rate with $\sigma_\mu^2 = 0.02$, $\phi_F = \phi_R = 0.85$ and $\rho_{R,F} = 0.5$.

Table 4: Stochastic properties of the empirical distribution of $\hat{\beta}_1$ under departure from rational expectations and constant term premium.

	Slope coefficient			Asymptotic t-ratio		
	2.5%	median	97.5%	2.5%	median	97.5%
$k_{1,2} = 1$						
$\hat{\beta}_1$	-0.0004	0.000	0.0005	-0.052	0.004	0.061
JB	0.058	(0.971)		0.101	(0.950)	
$k_{1,2} = 0.95$						
$\hat{\beta}_1$	0.063	0.160	0.456	0.341	0.476	0.851
JB	249.37	(0.000)		353.57	(0.000)	
$k_{1,2} = 0.9$						
$\hat{\beta}_1$	0.190	0.432	0.782	0.599	0.934	1.757
JB	21.338	(0.000)		288.53	(0.000)	
$k_{1,2} = 0.85$						
$\hat{\beta}_1$	0.344	0.645	0.897	0.888	1.445	2.757
JB	7.972	(0.018)		264.17	(0.000)	

Notes: The data-generating mechanism is the random walk vector ARMA components model fitted to spot and 3-month forward interest rate with $\sigma_F^2 = 0$, $\sigma_R^2 = 0$, $\phi_R = \phi_F = 0.2$ and $\sigma_\mu^2 = 0.02$.

4 The relationship between co-integration and VECM

While the relationship between co-integration and Single Equation models was inspected in Section 3, this section sheds light on the linkage between co-integration and the VECM technique. The analysis is carried out setting $m=3$ and $n=6$ and assuming co-integration, as defined by eqs.(9)-(10) (note that the same intuition carries over to $m=6$ and $n=12$ as well).

Given the existence of co-integration between $F_3^6(t)$ and $R_3(t+3)$, it becomes possible to run the following linear regression:¹⁶

$$F_3^6(t) = m + \gamma_0 R_3(t+3) + \xi(t) \quad (26)$$

where the $\xi(t)$ series is presumed to be white noise. Eq.(26) can be re-parameterized into the error correction mechanism (ECM) form replacing $F_3^6(t)$ by $F_3^6(t-1) + \Delta F_3^6(t)$ and $R_3(t+3)$ by $R_3(t+2) + \Delta R_3(t+3)$. The result is:

$$\Delta F_3^6(t) = m + \gamma_0 \Delta R_3(t+3) - [F_3^6(t-1) - \gamma_0 R_3(t+2)] + \xi(t). \quad (27)$$

¹⁶When $F_3^6(t)$ and $R_3(t+3)$ are co-integrated OLS suffers from second-order asymptotic bias and t -ratios are not asymptotically standard normal. Thus, to estimate γ_0 more sophisticated techniques such as Stock and Watson's (1993) dynamic OLS and dynamic GLS must be employed.

The ECM formulation shows that current changes in $F_3^6(t)$ is defined by the sum of two components. The first is proportional to the current change in $R_3(t+3)$, and the second is a partial correction for the extent to which $F_3^6(t-1)$ deviates from the equilibrium value corresponding to $R_3(t+2)$. This deviation is the ECM and it is shown by the term in square brackets.

The ECM formulation is important for this analysis because of the Granger Representation Theorem which states that for any set of I(1) variables, error correction (as defined by eq.(27)) and co-integration are equivalent representations (see Engle and Granger (1987)). Substituting into eq.(27) the stochastic processes which govern the forward and future spot interest rate one obtains:

$$\begin{aligned} \Delta x_{F_3^6}(t) + \epsilon(t) = m + \gamma_0[\Delta x_{R_3}(t+3) + k_{1,2}\epsilon(t)] + \\ -\{\mu(t-1) + x_{F_3^6}(t-1) - \gamma_0[k_{1,2}\mu(t-1) + x_{R_3}(t+2)]\} + \xi(t). \end{aligned} \quad (28)$$

Eq.(28) shows that both the R.H.S. and the first term of the L.H.S. are stationary processes. In addition $\xi(t)$ is a white noise error which is also stationary. Since a non stationary term cannot equal a stationary process, the term in curly brackets (i.e. the ECM)

$$\begin{aligned} -\{\mu(t-1) + x_{F_3^6}(t-1) - \gamma_0[k_{1,2}\mu(t-1) + x_{R_3}(t+2)]\} = \\ \Delta x_{F_3^6}(t) + \epsilon(t) - m - \gamma_0[\Delta x_{R_3}(t+3) + k_{1,2}\epsilon(t)] - \xi(t) \end{aligned} \quad (29)$$

must be stationary too. Since this term contains an I(1) process, the only value of the parameter γ_0 which guarantees stationarity is $\gamma_0 = 1/k_{1,2}$. This result explains how the presence of co-integration, as defined by eqs.(9)-(10), is detected by the VECM. Moreover, this result helps explain why, overall, tests based on the VECM methodology have greater tendency to support the EHTS than those based on Single Equation models. In fact, assuming rational expectations (i.e. $k_{1,2} = 1$) and time varying term premium, the VECM detects the co-integrating vector $[1 \ -1]$ fully consistent with the EHTS. On the contrary, the single equation parameter given by eq.(18) can capture both the presence of rational expectations and time varying term premium through, respectively, $k_{1,2}$ and the parameters which characterize the stochastic processes $x_{F_{n-m}^n}(t)$ and $x_{R_m}(t)$. In other words, while the former captures only the long run relationship between forward and spot rates but ignores the presence of time varying term premium, the latter can capture both these effects and, as a result, can lead to more frequent rejections of the EHTS.

5 A model for spot and forward interest rates

To comply with the condition of co-integration set out in Section 2, a simple parametric permanent-transitory components model, which encompasses the possibility that forward and future spot interest rates are driven by a common

stochastic trend, is employed. Similar two-component models have been employed in many different fields of macroeconomics and finance. Campbell and Shiller (1988), Fama and French (1988) and Summers (1986), for instance, make use of a two-component model to describe the evolution of stock prices where the random walk represents the rationally expected present value of future dividends (the fundamentals solution), and the transitory components represent price "fads". Clark (1987, 1989), and Nelson and Plosser (1982), employ a version of the two-components model to evaluate the relative importance of the stochastic trend and the stationary cyclical components of economic activity. Hai et al. (1997), employ a two-component model to estimate term premia in foreign exchange markets, where spot and forward exchange rates are driven by a common random walk which represents the long-run equilibrium.

The motivation for the empirical analysis carried out in this section is that by making use of a permanent-transitory components model, it becomes possible to estimate the parameters that define the co-integrating stochastic process of spot and forward rates. Such estimates, in turn, can be employed to shed light on the relationships between co-integration and Single Equation models set out in Section 3, as well as to measure the contributions of rational expectations and time varying term premium to departures from the EHTS. A second motivation for this empirical analysis can be drawn on Cox et al. (1985) and Campbell and Shiller (1987) that show that the time series behavior of each yield to maturity is driven by a common driving force (see also, among others, Hall et al. (1992) and Engsted and Tanggaard (1994)). Setting $n = 6$ and $m = 3$ to simplify the notation, the considerations set out in Section 2 lead to the following state space model for spot and forward interest rate (the same intuition carries over to $m=6$ and $n=12$):¹⁷

$$\begin{bmatrix} F_3^6(t) \\ R_3^e(t+3) \end{bmatrix} = \begin{bmatrix} k_{1,2} & 1 & 0 \\ 1 & 0 & \phi_R^3 \end{bmatrix} \cdot \begin{bmatrix} \mu^*(t) \\ x_{F_3^6}(t) \\ x_{R_3}(t) \end{bmatrix} \quad (30)$$

$$\begin{bmatrix} \mu^*(t) \\ x_{F_3^6}(t) \\ x_{R_3}(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_F & 0 \\ 0 & 0 & \phi_R \end{bmatrix} \cdot \begin{bmatrix} \mu^*(t-1) \\ x_{F_3^6}(t-1) \\ x_{R_3}(t-1) \end{bmatrix} + \begin{bmatrix} \epsilon(t) \\ \epsilon_{F_3^6}(t) \\ \epsilon_{R_3}(t) \end{bmatrix} \quad (31)$$

$$\begin{bmatrix} \epsilon(t) \\ \epsilon_{F_3^6}(t) \\ \epsilon_{R_3}(t) \end{bmatrix} \sim iid N \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\mu^2 & 0 & 0 \\ 0 & \sigma_F^2 & \sigma_{F,R} \\ 0 & \sigma_{F,R} & \sigma_R^2 \end{bmatrix} \right) \quad (32)$$

where $R_3^e(t+3)$ is the expectations at time t of the three month spot interest rate in three period, i.e. $E_t[R_3(t+3)]$. The model above is a bivariate model

¹⁷In line with the theory of state space models, the measurement equation does not necessarily have to be a stochastic equation. This happens when their disturbance terms are defined in the transition equation like in the system given by eqs.(30)-(32).

which originates from the state space representation of eqs.(5)-(6) and (12)-(11). To gain some insight into the model's ability to account for the data the special case where the transient components follows univariate AR(1) processes is examined - a simplification that allows the interpretation of the analytic formulae set out in Section 3. To avoid identification problems the covariance $\sigma_{F,R}$ in eq.(32) is set equal to zero.¹⁸

6 Data

The dataset employed begins in January 1964 and extends in May 2000 for the three, six and twelve months US Treasury Bill rates (average auctions). These data are taken from the FRED database at the Federal Reserve Bank of St. Louis. The forward interest rates $F_3^6(t)$ and $F_6^{12}(t)$, are the rates implicit in the yield curve extracted using the twelve, six and three month rates. The observed yield on each bill has been derived from the price of that bill on a given day (last trading day of the month) so that the data relate to bills which are identical in all respects other than term.

The sample cover three monetary regimes which are distinguished by the degree of interest rate targeting undertaken by the Federal Reserve. The first regime, covering the period up to September 1979, corresponds to a period during which the Federal Reserve was targeting interest rates. The period from October 1979 to September 1982 covers the Federal Reserve's "new operating procedures", when it ceased targeting interest rates in favor of monetary aggregates. The final regime, from October 1982 onward, corresponds to the abandonment of the "new operating procedures" and the resumption of partial interest rates targeting. Plots of the yield data are provided in Figures 2 and 3 which illustrates the similar behavior of the yields over the sample period. In particular, the two figures illustrate that the yields were considerably more volatile during the "new operating procedures" regime than they have been at other times.

6.1 Maximum likelihood estimates

The two-component model for spot and forward interest rates is estimated using kalman filter and maximum likelihood. All the maximum likelihood estimations are worked out using the BFGS (Broyden, Fletcher, Goldfarb, Shanno) algorithm in Gauss.¹⁹

¹⁸The identification issue is typically encountered when a nonstationary time series is decomposed into a stochastic trend and a stationary component. It arises because there exist more parameters in the structural model given by eqs.(30)-(32) than in the reduced form model. Unless an identifying restriction is imposed in the structural model, leaving the same number of parameters in both models, the decomposition is not possible. See Nelson and Plosser (1982) for further details.

¹⁹Once the final estimates have been obtained, a sensitive analysis to check the stability of the estimates has been conducted. This analysis consisted of feeding the BFGS algorithm with the final estimates obtained from the previous stage and to check that it delivered estimates consistent with those previously obtained.

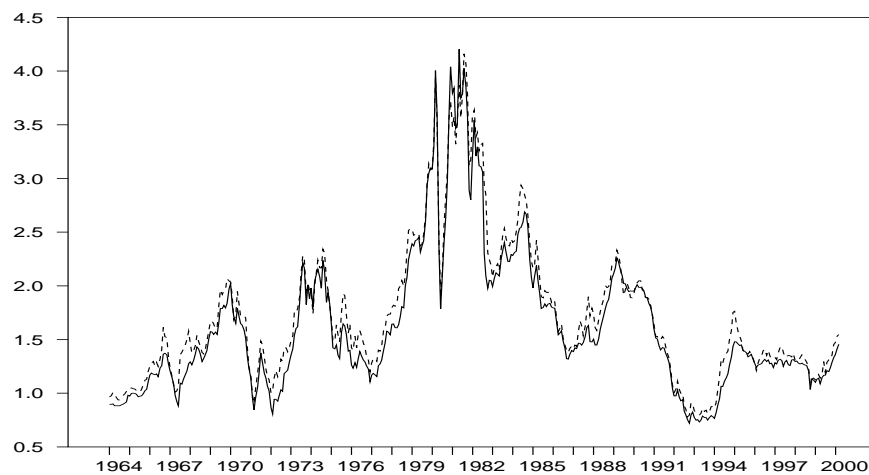


Figure 2: Three month spot (solid line) and forward (dotted line) interest rates for the period 1964:01-2000:06.

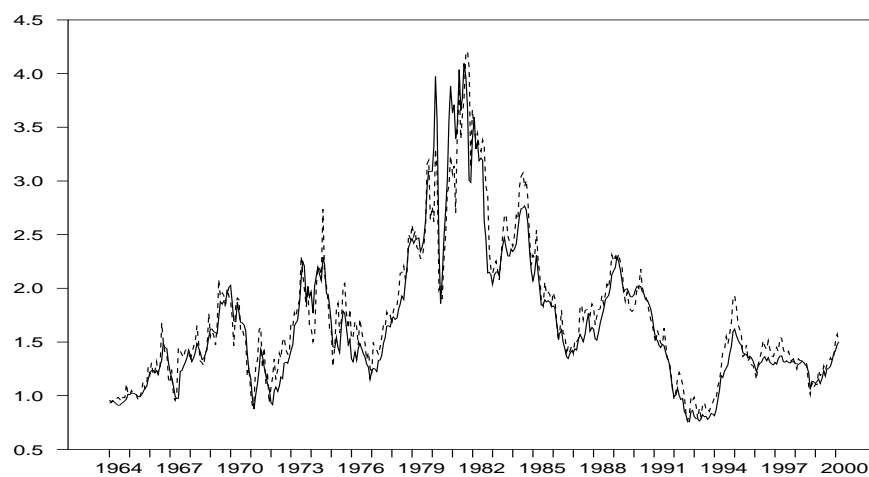


Figure 3: Six month spot (solid line) and forward (dotted line) interest rates for the period 1964:01-2000:06.

Table 5 reports the maximum likelihood estimates and asymptotic standard errors from the 3-month spot and forward rates system, while Table 6 reports estimates from the 6-month spot and forward rates. The top panels of both the tables set out estimates for the unrestricted model while restrictions to the model are tested in the second, third and bottom panels.

To check on the adequacy of the specification, the Ljung and Box (1978) portmanteau test is applied to the vector of residuals of the ARMA model, as proposed in Lütkepohl (1993). The test statistic denoted by $Q(p)$ is computed using the sample autocorrelation matrix of the model residuals, where p is the number of residual sample autocorrelation used. Under the null hypothesis that the model is correctly specified, $Q(p)$ has an asymptotic χ^2 distribution with the degree of freedom equal to n^2p minus the number of estimated coefficients in the vector ARMA, where n is the number of equations. Tables 5 and 6 report $Q(12)$ and $Q(24)$ along with their associated p-values, and show that the null cannot be rejected at the 5% level for both the models. Overall these results seem to suggest that the models are reasonably well specified.

The asymptotic standard errors are generally small, relative to the point estimates, suggesting that the parameters are precisely estimated. The estimated values of $k_{1,2}$ turn out to be very close to one signalling moderate departures from the rational expectations hypothesis. The parameters related to the term premium are all statistically significant, highlighting the presence of time varying term premium. This, in turn, suggests that rejections of the EHTS might be caused by both departures from rational expectations and time varying term premium, whereas the role played by the term premium seems to be more important. The estimates also indicate that forward and spot interest rate variability is not dominated by the random walk component. In fact, for both Tables 5 and 6 the estimated standard deviation of the random walk innovations is lower than the standard deviations of the transitory innovations. Thus, interest rate dynamics are not dominated by unpredictable changes in the permanent component. These results, in turn, suggest that macroeconomic models can be helpful to explain the short end of the term structure.

Standard likelihood-ratio tests are employed to test the null hypotheses of rational expectations (i.e. $k_{1,2} = 1$), absence of time varying term premium (i.e. $\phi_F = \phi_R = 0$) as well as the joint hypothesis of rational expectations and absence of term premium.²⁰ The latter, in turn, amounts to testing for the EHTS. These tests are reported respectively in the second, third and bottom panels of Table 5 and 6. Marginal significance levels (p-values) of the likelihood-ratio test statistics indicate that the null of rational expectations cannot be rejected and that the EHTS is rejected solely because of the presence of time varying term premium. Such evidence holds for both three and six-month spot and forward interest rates. Thus, recalling that the EHTS is a joint hypothesis of rationality and risk neutrality, the empirical results set out in this section support the

²⁰The likelihood-ratio tests are based on the result that, under the null, the statistic $LR = -2\log(L_R/L_{UR})$ is asymptotically distributed as χ_m^2 where m is the number of restrictions and where L_R and L_{UR} are the values of the likelihood functions under the restricted and unrestricted case.

majority of the literature which has inspected the hypothesis assuming that the rational expectation leg of the theory holds and testing for its second leg, i.e. the presence of time varying term premium (see, for instance, Lee (1995), Iyer (1997), Engle et al. (1987), Hall et al. (1992), Hejazi et al. (2000) and Gravelle and Morley (2005)).

Table 7 displays various population moments implied by the point estimates as well as sample estimates. The implied moments have been calculated by mean of eqs.(18) and (20). As previously explained, to avoid problems of identification in the estimated model given by eqs.(30)-(32) the contemporaneous correlation between the transitory component's innovations of both spot and forward interest rates was restricted to zero. When, however, it comes to the computation of the different moments, such restriction produces values quite different from the sample estimates. Since one possible explanation of the gap between implied and sample moments is that it might be caused by the above restriction, new computations of the implied moments are carried out, this time relaxing the restriction. Once the restriction is relaxed, it can be shown that the implied moments match up to the sample moments for positive values of the correlation. The closest distance between implied and sample moments are obtained when the correlation is set equal to 0.9. These values are reported in Table 7.

Under the "eyeball" metric, the model does quite a fair job of matching these moments. The average distances between sample and implied moments is equal to 30%.²¹ The implied slope coefficients β_1 match up quite well with the sample OLS estimates, being the distances equal to 26% for the three-month and 14% for the six-month spot and forward rates. The next section provides evidence that both the implied and the sample values originate from the same probability distribution.

7 Simulations

This section tests whether the estimated two-component models can match important functions of the data that were not implicitly imposed in estimation. In particular, attention is focused on the ability of the model to match those features of the data reported in Table 1. Such tests are carried out by generating simulation distributions of the slope coefficient estimators, where the data generating process is the two-component model with parameter values equal to the point estimates of Tables 5 and 6.

These distributions are built from simulations of 10,000 trials where for each trial i ($i = 1, 2, \dots, 10,000$) a scalar sequence of observations $[(\epsilon^i(t), \epsilon_{F_{n-m}}^i(t), \epsilon_{R_m}^i(t))]'_{t=1}^T$ is generated from normal distributions with mean 0 and variance, respectively, $\hat{\sigma}_\mu^2$, $\hat{\sigma}_F^2$ and $\hat{\sigma}_R^2$. The sequences of observations $[(\mu^i(t), x_{F_{n-m}}^i(t), x_{R_m}^i(t))]'_{t=1}^T$ are, then, generated according to eqs.(9)-(10) and then combined to construct sequences of spot and forward rates, $[(R_m^i(t), F_{n-m}^{n,i}(t))]'_{t=1}^T$ for both $n = 6$ and

²¹The distance between sample and implied moments is computed taking the difference between the two and dividing by the value of the implied moment.

Table 5: Maximum likelihood estimates of the permanent-transitory components model for three-month spot and forward interest rates.

ϕ_F	ϕ_R	$k_{1,2}$	σ_μ	σ_F	σ_R
Unrestricted					
log likelihood = 732.65					
0.780 (0.055)	- (-)	1.059 (0.040)	0.063 (0.007)	0.160 (0.007)	0.184 (0.020)
- (-)	0.888 (0.033)	Q(12)=51.33 p-value=0.208 Q(24)=94.19 p-value=0.417			
Restriction: $k_{1,2} = 1$					
log likelihood = 731.6					
0.804 (0.056)	- (-)	- (-)	0.062 (0.008)	0.162 (0.007)	0.176 (0.018)
- (-)	0.903 (0.032)	LR=2.100 (0.147)			
Restriction: $\phi_F = \phi_R = 0$					
log likelihood = 454.6					
- (-)	- (-)	1.035 (0.009)	0.144 (0.005)	0.339 (0.012)	13×10^{-4} (0.005)
- (-)	- (-)	LR=556.1 (0.000)			
Restriction: $\phi_F = \phi_R = 0 \cap k_{1,2} = 1$					
log likelihood = 447.6					
- (-)	- (-)	- (-)	0.144 (0.005)	0.345 (0.012)	3×10^{-4} (0.014)
- (-)	- (-)	LR=570.1 (0.000)			

Notes: Asymptotic standard errors in parentheses. LR is likelihood ratio statistic. $LR \sim \chi_m^2$ where m is the number of restrictions. $Q(p)$ are p^{th} order Ljung-Box statistics for serial correlation. $Q(12) \sim \chi_{44}^2$ and $Q(24) \sim \chi_{92}^2$.

Table 6: Maximum likelihood estimates of the permanent-transitory components model for six-month spot and forward interest rates.

ϕ_F	ϕ_R	$k_{1,2}$	σ_μ	σ_F	σ_R
Unrestricted					
log likelihood = 739.67					
0.879 (0.052)	- (-)	1.029 (0.075)	0.045 (0.011)	0.173 (0.007)	0.186 (0.029)
-	0.941	Q(12)=41.54 p-value=0.577			
(-)	(0.026)	Q(24)=75.21 p-value=0.898			
Restriction: $k_{1,2} = 1$					
log likelihood =739.59					
0.883 (0.051)	- (-)	-	0.045 (0.011)	0.173 (0.007)	0.184 (0.006)
-	0.942				
(-)	(0.026)	LR=0.161 (0.689)			
Restriction: $\phi_F = \phi_R = 0$					
log likelihood =400.62					
- (-)	- (-)	0.998 (0.010)	0.137 (0.005)	0.406 (0.014)	1×10^{-5} (0.001)
-	-				
(-)	(-)	LR=678.1 (0.000)			
Restriction: $\phi_F = \phi_R = 0 \cap k_{1,2} = 1$					
log likelihood =400.61					
- (-)	- (-)	- (-)	0.137 (0.005)	0.406 (0.014)	2×10^{-4} (0.011)
-	-				
(-)	(-)	LR=678.1 (0.000)			

Notes: Asymptotic standard errors in parentheses. LR is likelihood ratio statistic. $LR \sim \chi_m^2$ where m is the number of restrictions. $Q(p)$ are p^{th} order Ljung-Box statistics for serial correlation. $Q(12) \sim \chi_{44}^2$ and $Q(24) \sim \chi_{92}^2$.

Table 7: Sample and implied moments from the maximum likelihood estimates of the permanent-transitory components model.

	Sample	Implied
$Cov[TP_{3,6}(t); fp_{3,6}(t)]$	0.007	0.019
$Var[fp_{3,6}(t)]$	0.011	0.037
β_1	0.673	0.531
$Var[TP_{3,6}(t)]$	n.a.	0.022
$Cov[TP_{6,12}(t); fp_{6,12}(t)]$	0.027	0.041
$Var[fp_{6,12}(t)]$	0.032	0.052
β_1	0.897	0.786
$Var[TP_{6,12}(t)]$	n.a.	0.040

$m = 3$ and $n = 12$ and $m = 6$. The computer-generated observations are then employed to estimate the slope coefficient in the regression of the expected excess return on forward premium, $\hat{\beta}_1^i$.

The 10,000 observations on $\hat{\beta}_1^i$ form the empirical distribution under the null hypothesis that the estimated permanent-transitory components model is the true data-generating mechanism.

7.1 Results

Table 8 reports the lower 2.5, 50 and 97.5 percentiles of the empirical distribution for $\hat{\beta}_1$. The top panel contains simulations for $n = 6$ and $m = 3$ while the bottom panel reports simulations for $n = 12$ and $m = 6$. P-values are the proportion of the empirical distribution that lies to the right of the values estimated from the data.

The table provides some interesting information about the sampling properties of the empirical distribution of $\hat{\beta}_1$. The median values of $\hat{\beta}_1$ are reasonably close to the implied population values. Both the sample estimates, which for $n = 6$ and $m = 3$ is 0.673, and for $n = 12$ and $m = 6$ is 0.897, lie to the right of the respective medians. Moreover, none of the individual p-values lie outside the interval $[0.025; 0.975]$. Hence, the hypothesis that the regression estimates of β_1 were drawn from the empirical null distributions cannot be rejected at the standard significance levels. Moreover, according to the Jarque-Bera tests the null that the probability distributions of $\hat{\beta}_1$ are normal cannot be rejected. Thus, the negligible (not statistically significant) departures from rational expectations highlighted in both Tables 5 and 6, do not impact on the probability distribution of $\hat{\beta}_1$, which remains normally distributed.²²

²²Remember that in Section 2 it was shown that when departure from rational expectations is introduced (i.e. when the parameter $k_{1,2}$ takes values different from 1) the probability distribution of $\hat{\beta}_1$ degenerates to non-normal distributions.

Table 8: Features of the empirical distribution of $\hat{\beta}_1$ for $n = 6$ and $m = 3$ and for $n = 12$ and $m = 6$.

Slope coefficient			
2.5%	median	97.5%	p-value
Three-month spot and forward interest rates			
0.502	0.595	0.691	0.054
JB=0.242 (0.885)			
Six-month spot and forward interest rates			
0.628	0.788	0.952	0.089
JB=0.179 (0.914)			

Notes: P-values are the proportion of the empirical distribution that lies above the values estimated from the data. The data generating process is the two-component model of eqs.(30)-(32) fitted to three-month and six-month spot and forward interest rates.

8 Implied time varying term premia

This section inspects the stochastic properties of the time varying term premium for both the three and six month forward interest rates. The two series are worked out making use of the kalman filter decomposition into permanent and transitory components and employing the term premium formulation. More specifically, estimations of the three and six month time varying term premium can be carried out imposing rational expectations (i.e. setting $k_{1,2} = 1$ into the state space model (30)-(32)) and employing eq.(19).²³ The two detected series are depicted in Figure 4 and show similar stochastic properties.

Table 9 reports the first four moments of the three and six month term premium distribution as well as Jarque-Bera tests for normality.²⁴ The null of normality is shown to be strongly rejected for both the series under analysis. The large values of the fourth moment of the distributions, in fact, signals that the probability density functions of the term premia are simultaneously "peaked" and have "fatter tails" than Normal distributions (i.e. they are leptokurtic). To further inspect the shape of the probability density functions, kernel density estimates are displayed in Figure 5.²⁵ This provides a first glance of the different moments of both the distributions which look symmetrical about zero. Such

²³Notice that eq.(19) is worked out setting $n = 6$ and $m = 3$. However, the same intuition carries over to $m=6$ and $n=12$.

²⁴Under the null hypothesis of normal distribution, the Jarque-Bera statistic follows a χ_k^2 distribution where k represents the number of estimated coefficients used to create the series.

²⁵The kernel density estimates can be viewed as an histogram where the "boxes" are replaced by "bumps" which are smoother. The kernel function used, which determines the shape of the bumps, is the Epanechnikov function.

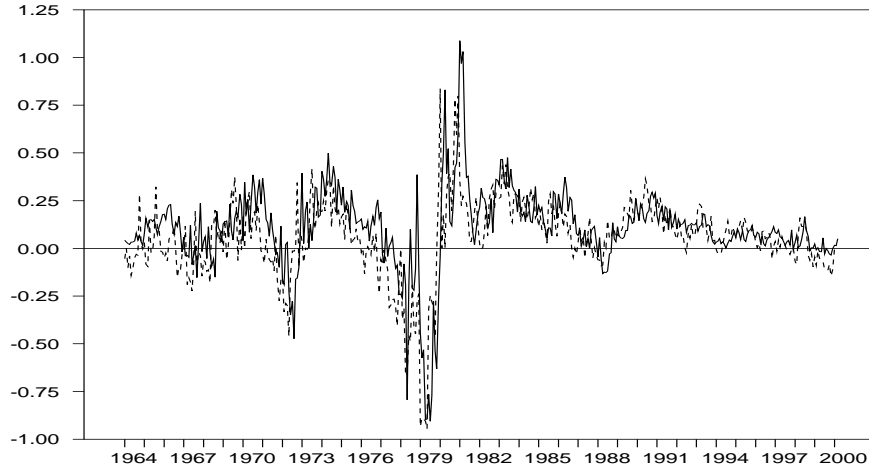


Figure 4: Three month (solid line) and six month (dotted line) time varying term premium for the period 1964:01-2000:03.

characteristic makes the two term premia compatible with theoretical distributions other than the Normal, Logistic and Student's t in particular. The null that the above term premia originate from Logistic or Student's t distributions, however, turns out to be strongly rejected when tested by means of the Cramer von Mises (C-M), the Anderson-Darling (A-D) tests and the Q-Q diagrams.²⁶ From Figure 4 the plotted term premia appear to be persistent and to fluctuate between positive and negative values. Both the series turn out to be reasonable in magnitude being, on average, equal to 6% and 4% of the level of forward rates. It is also visually apparent that the stochastic properties of the three and six month term premium are, all in all, similar. The contemporaneous correlation between the two series is, in fact, 0.638.

Table 10 computes ADF and Phillip-Perron (PP) tests for the presence of a unit root, while Table 11 reports the AC function (ACF) and the PAC function (PACF). In line with previous studies, the presence of a unit root in the stochastic processes governing the term premia is strongly rejected (see, for instance, Engle et al. (1987) and Hall et al. (1992)). Moreover, according to both the ACF and PACF, the term premia appear to be governed by similar AR(p) stochastic processes with $p \in [1, 3]$. Both the three and six month term premia are negative for much of the latter 1970s (a period characterized by economic expansion), and positive during the 1963, 1973, 1980 and 1981 contractions, while they take values close to zero during the final six years of the sample. Furthermore, the term premium at both maturities heightened in correspondence to the 1979 change in the FED operating procedures. The latter is a

²⁶Both the C-M and the A-D tests as well as the Q-Q diagrams are not reported to economize on space.

Table 9: Statistics of the three and six month time varying term premium.

	$TP_3^6(t)$	$TP_6^{12}(t)$
Sample Mean	0.087	0.042
SD	0.169	0.158
Skewness	-0.444	0.118
Kurtosis	9.630	8.294
J-B	811.1	505.6
	(0.000)	(0.000)

Notes: p-values in parenthesis. $TP_3^6(t)$ estimated for the period 1964:01-2000:03 while $TP_6^{12}(t)$ for the period 1964:01-1999:12.

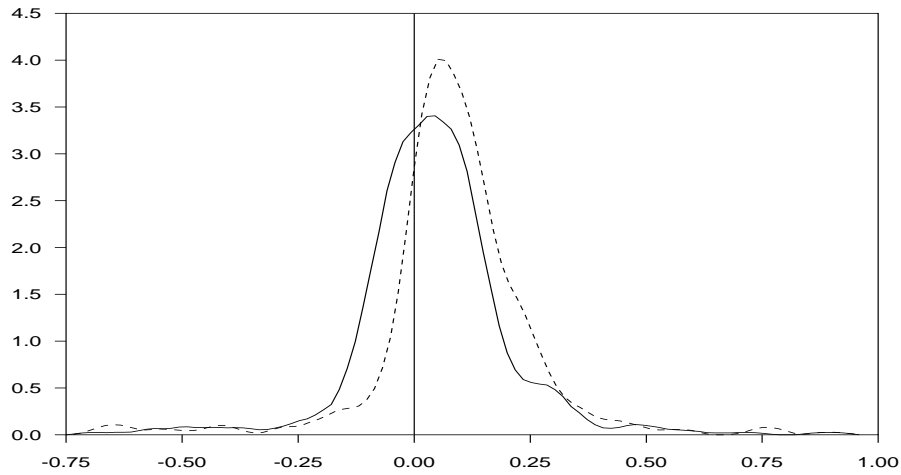


Figure 5: Kernel density estimates of the three month (dotted line) and six month (solid line) time varying term premium for the period 1964:01-2000:03.

stylized fact which has been detected by many other empirical models of term premium in the term structure (see, for instance Iyer (1997), and Engle et al. (1987)).²⁷

Time varying term premium can emerge for a variety of reasons. One hypothesis that is frequently suggested is that it represents a rational risk premium. For

²⁷The change in the FED operating procedure coincides with the decision at the October 6, 1979 FOMC meeting to switch the focus of monetary policy from targeting interest rates to tighter control of the monetary base, in an effort to bring down the high inflation that the US experienced during the late 70s.

Table 10: Unit root tests on the three and six month time varying term premium.

	$TP_3^6(t)$	$TP_6^{12}(t)$
ADF	-4.692 (0.000)	-6.773 (0.000)
PP	-6.638 (0.000)	-8.416 (0.000)

Notes: Lag length for the ADF regressions is chosen following the Schwartz Criterion while for the P-P it is fixed to 5. P-values in parenthesis.

instance, by employing standard intertemporal asset pricing models of the term structure under rational expectations, it is possible to show that the risk premium is proportional to the conditional covariance of the intertemporal marginal rate of substitution to consumption, and the profit from interest rate speculation. According to this framework, the conditional covariance can depend on sample information and need not be time invariant. Hence, the risk premium can fluctuate over time, and assume positive and negative values depending on the sign of such covariance.²⁸

9 Conclusions

The contribution of this paper to the literature on the term structure of interest rates is twofold. First, it bridges the two strands of literature based on co-integration and Single Equation models. Introducing co-integration into the stochastic processes of spot and forward interest rates, it is shown that it becomes possible to separately model departures from rational expectations and time varying term premium. Thus, making use of the moving average representation of the parameters of the regressions based on forward-spot spreads and simulations, the paper measures the separate effect of departures from rational expectations and time varying term premium on the parameters themselves. Moreover, the paper shows that a necessary condition for the EHTS to hold, is that co-integration between spot and forward rates occurs with a co-integrating vector equal to $[1 \ -1]$.

The paper also sheds light on the linkage between Single Equation model and ECM representation of spot and forward interest rates. In particular, it shows that the ECM only captures the impact of rational expectations on the EHTS by means of the long run relationship between forward and spot rates, while the presence of time varying term premium is not accounted for. On the contrary, it is shown that Single Equation models can capture both the effects. This, in

²⁸Lee (1995), for instance, makes use of an intertemporal consumption-based asset pricing model to show that the risk premium arises as compensation for the covariance risk between output and money supply (see also Engle et al. (1987) and Campbell et al. (1997)).

Table 11: ACF and PACF for the three and six month time varying term premium.

Lag	$TP_3^6(t)$		$TP_6^{12}(t)$	
	ACF	PACF	ACF	PACF
1	0.760*	0.760*	0.684*	0.684*
2	0.627*	0.118*	0.542*	0.140*
3	0.555*	0.108*	0.413*	-0.002
4	0.447*	-0.069	0.349*	0.060
5	0.327*	-0.100	0.293*	0.023
6	0.246*	-0.019	0.289*	0.090
7	0.197*	0.030	0.191*	-0.122
8	0.163*	0.046	0.136*	-0.027
9	0.165*	0.092	0.170*	0.154*
10	0.137*	-0.041	0.158*	-0.009
11	0.122*	-0.001	0.158*	0.016
12	0.065	-0.133*	0.064	-0.163*

Notes: * (**) significant at 5% (1%). $TP_3^6(t)$ estimated for the period 1964:01-2000:03 while $TP_6^{12}(t)$ for the period 1964:01-1999:12.

turn, explains why tests based on the VECM methodology have greater tendency to support the EHTS than those based on Single Equation models.

The second aim of this paper, was to estimate the separate contribution of departures from rational expectations and time varying term premium to the invalidation of the EHTS making use of real data. Throughout the analysis, a structural time series approach is adopted in an effort to further understand the dynamics of both three and six month term premia. The two-component model employed makes possible the disentangling of the relative contribution of departures from rational expectations and time varying term premium to the invalidation of the EHTS. The model draws its motivation from studies which show that the time series behavior of each yield to maturity is driven by common driving forces. Standard diagnostic tests and a simulation experiment were carried out to gauge the adequacy of the representation.

The empirical results show that departures from rational expectations are strongly rejected and that rejections of the EHTS are solely due to the presence of time varying term premium. Such results hold for both the three and six month term premium.

Estimates of the three and six month term premia under rational expectations are worked out. Both the series turn out to be reasonable in magnitude, persistent, fluctuate between positive and negative values, and show similar stochastic properties.

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